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Shape sensitivity analysis for nonlinear steady-state heat conduction problems

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Abstract—In this paper the Kirchhoff transformation for shape sensitivity analysis of bodies with steady-state non-linear heat conduction is considered. The approach reduces the non-linear thermal problem to the standard Laplace problem. The variation of a general integral functional is described in terms of adjoint quantities. The design sensitivities are calculated using the material derivative concept. A thermal diffuser problem is considered to illustrate the method proposed. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

In solving problems of nonlinear thermomechanics the important step is to develop methods for efficient assessment of the nonlinear response of the body subject to thermal loading. On the other hand, whenever a physical response is to be calculated from a mathematical model there is also an interest in the sensitivity of that response with respect to parameters (design variables) of the problem. Therefore, techniques of the so-called design sensitivity analysis (DSA) have been developed to calculate variations in the response quantities with respect to some design variables. The sensitivity information may be used to assess the effect of uncertainties in the mathematical model, to predict the change in the response due to a design-related change in the parameters, and to optimize the system with the aid of mathematical optimization techniques. The methods of sensitivity analysis have been explored in various fields of science and engineering in the last two decades or so. More recently, the growing interest in the optimum design of systems subject to temperature constraints can be observed. The review of DSA methods for linear thermal problems can be found in refs. [1–3, 8], for instance. Although effective computer implementations of DSA in general-purpose finite element codes for this class of thermal problems are still rather rare, the research area can now be considered well established. This is by no means so with DSA applications to nonlinear thermal problems. Even though the literature here is already quite extensive, no single formulation appears to be widely accepted as the most effective [4–7, 9–10].

The purpose of this paper is to present and discuss a further alternative for solving shape (and non-shape) sensitivity problems of temperature dependent, steady-state, isotropic heat conducting solids. The methodology is quite straightforward: (i) the nonlinear problem is first reduced to a linear one in the form of the standard Laplace problem by using the Kirchhoff transformation; (ii) the resulting problem is solved by using a version of the technique known for the linear thermal problems; and (iii) the sensitivity solution so obtained is transformed back to the original problem. The shape sensitivity solution for the linear problem will be obtained by combining the so-called adjoint system method (ASM) with the so-called material derivative (MD) concept. The ASM consists in forming an adjoint system corresponding to the response functional at hand and using the adjoint fields to obtain the sensitivity desired. The MD technique is one of the two frequently adopted methodologies for solving shape sensitivity problems. In it, the material derivative concept of continuum mechanics is used to obtain variations of the field variables. Also, variations to the volume and surface integrals over a variable domain are used to obtain the design sensitivity expression for the response functional.

According to the above comment only the ASM approach is used in this paper, although an alternative technique known as the direct differentiation method (DDM) could in principle be employed as well. A quite similar remark can be made with respect to the

NOMENCLATURE

T	temperature	$\dot{D} = \mathbf{V}\cdot\mathbf{n}$	
k	thermal conductivity	H	curvature of the boundary
E	Euclidean space	J	functional
$\mathbf{q}$	heat flux	$H^1(\Omega)$	Sobolev space of order 1.
$\mathbf{n}$	unit vector normal to $\partial\Omega$		
$\mathbf{V}$	vector of a deformation field	Greek symbols	
t	parameter	Ω	region considered
$\mathbf{x}$	coordinate vector	$\partial\Omega$	boundary surface.

MD approach—the alternative technique known as the control volume (CV) approach could also be used.

PRIMARY EQUATION

We consider the nonlinear steady-state heat conduction problem described in a three-dimensional region Ω by the equation

$$\frac{\partial}{\partial x_1}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial x_2}\left(k\frac{\partial T}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(k\frac{\partial T}{\partial x_3}\right)=$$

$$\nabla(k\nabla T) = 0 \quad (x_1,x_2,x_3)\in\Omega\subset E \tag{1}$$

in which T is the temperature and $k = k(T)$ is the temperature dependent thermal conductivity. The boundary conditions for equation (1) are given as

$$T = \hat{T} \quad (x_1,x_2,x_3)\in\partial\Omega_T \tag{2}$$

$$\mathbf{q} = -k\frac{\partial T}{\partial \mathbf{n}} = \hat{\mathbf{q}} \quad (x_1,x_2,x_3)\in\partial\Omega_q, \tag{3}$$

where $\hat{T}$ is a given temperature on the boundary surface $\partial\Omega_T$ with specified temperature, $\mathbf{q}$ is the heat flux, $\mathbf{n}$ is the unit outward vector normal to $\partial\Omega$ and $\hat{\mathbf{q}}$ is a given heat flux on the boundary surface $\partial\Omega_q$.

In order to transform equation (1) into the standard Laplace problem the Kirchhoff transformation can be used. The temperature T is replaced in it by a new variable $\vartheta = \vartheta(T)$ such that

$$\vartheta(T) = \frac{1}{k_0}\int_{T_0}^{T} k(T)\mathrm{d}T \quad k(T) = k_0\frac{\partial\vartheta}{\partial T}, \tag{4}$$

where $k_0 = k(T_0)$. By using equation (4) in equation (1) we obtain

$$\nabla^2\vartheta = 0 \quad \text{in}\ \Omega \tag{5}$$

with the boundary conditions

$$\vartheta = \hat{\vartheta} = \frac{1}{k_0}\int_{T_0}^{\hat{T}} k(T)\mathrm{d}T \quad \text{on}\ \partial\Omega_T, \tag{6}$$

$$\frac{\partial\vartheta}{\partial\mathbf{n}} = \frac{\partial\vartheta}{\partial T}\frac{\partial T}{\partial\mathbf{n}} = k\frac{\partial T}{\partial\mathbf{n}} = -\hat{\mathbf{q}} \quad \text{on}\ \partial\Omega_q. \tag{7}$$

MATERIAL DERIVATIVE CONCEPT

A one parameter family of perturbed domains may be defined by the mapping

$$\mathbf{x}_t = \mathbf{x}+t\cdot\mathbf{V}(x), \tag{8}$$

where $\mathbf{V}(x)$ is the vector of a deformation field and t is the parameter. Let z_t be the solution of a boundary value problem

$$Az_t = f \quad \mathbf{x}\in\Omega_t \tag{9}$$

$$Az_t = g \quad \mathbf{x}\in\partial\Omega_t. \tag{10}$$

Then $z_t(x_t) = z_t(\mathbf{x}+t\mathbf{V}(x))$ is the solution of the boundary value problem in Ω_t and it is evaluated at a point $\mathbf{x}_t$ that moves with t.

Material derivative $\dot{z}$ at $\mathbf{x}$ is defined as

$$\lim_{t\to 0}\left|\frac{z_t(\mathbf{x}+t\mathbf{V}(x)-z(x)}{t}-\dot{z}(x)\right| = 0. \tag{11}$$

Then

$$\dot{z}(x) = z'(x)+\nabla z\mathbf{V}(x), \tag{12}$$

where

$$z'(x) = \lim_{t\to 0}\left|\frac{z_t(x)-z(x)}{t}\right|. \tag{13}$$

The variation of the functional defined in the domain Ω

$$J_\Omega = \int_\Omega F(z)\,\mathrm{d}\Omega \tag{14}$$

is given as

$$\delta J_\Omega = \int_\Omega F_{,z}z'\,\mathrm{d}\Omega+\int_{\partial\Omega} F\dot{D}\,\mathrm{d}(\partial\Omega) \tag{15}$$

and the variation of the functional defined on the boundary $\partial\Omega$

$$J_{\partial\Omega} = \int_{\partial\Omega} G(z)\,\mathrm{d}(\partial\Omega) \tag{16}$$

is given as

$$\delta J_{\partial\Omega} = \int_{\partial\Omega} (G_{,z} z' + (\nabla G \mathbf{n} + GH)\dot{D})\, \mathrm{d}(\partial\Omega) + (V_1 n_2 - V_2 n_1) G(z) \,|_{\mathrm{B}}^{\mathrm{A}} \quad (17)$$

where $F_{,z}$, $G_{,z}$ denote the partial derivatives of F and G with respect to z, $\dot{D} = \mathbf{V}\cdot\mathbf{n}$, H is the curvature of the boundary $\partial\Omega$, $\mathbf{V}$, is the velocity field, and A and B are the end points of boundary $\partial\Omega$.

ADJOINT AND SENSITIVITY EQUATIONS

Let us now consider the problem of a functional J of the form

$$J = \int_\Omega f(\vartheta)\, \mathrm{d}\Omega + \int_{\partial\Omega} g\left(\vartheta, \frac{\partial \vartheta}{\partial n}\right) \mathrm{d}(\partial\Omega), \quad (18)$$

where f and g are regular functions that are continuous and differentiable with respect to their arguments. The functional depends on the variable ϑ in Ω and on ϑ and its normal derivative on the boundary $\partial\Omega$. Variational form of the state equation (5) $\div$ (7) is given as

$$\int_\Omega \nabla\vartheta\nabla\eta\, \mathrm{d}\Omega + \int_{\partial\Omega} q\eta\, \mathrm{d}(\partial\Omega) + \int_{\partial\Omega} \vartheta_{,n}\eta\, \mathrm{d}(\partial\Omega) = 0 \quad \text{for all } \eta \in K, \quad (19)$$

where $K = \{\eta, \eta \in H^1(\Omega)\}$ is the admissible set for the problem and $H^1(\Omega)$ is the Sobolev space of order 1.

By subtracting equation (19) from equation (18) one obtains an augmented functional as

$$\bar{J} = \int_\Omega f(\vartheta)\, \mathrm{d}\Omega + \int_{\partial\Omega} g(\vartheta, \vartheta_n)\, \mathrm{d}\Omega - \int_\Omega \nabla\vartheta\nabla\eta\, \mathrm{d}\Omega - \int_{\partial\Omega} q\eta\, \mathrm{d}(\partial\Omega) - \int_{\partial\Omega} \vartheta_{,n}\eta\, \mathrm{d}(\partial\Omega). \quad (20)$$

By using equations (15) and (17) we can obtain variation of the augmented functional as

$$\begin{aligned}
\delta\bar{J} = {} & \int_\Omega f_{,\vartheta}\vartheta'\, \mathrm{d}\Omega + \int_{\partial\Omega} f\dot{D}\, \mathrm{d}(\partial\Omega) \\
& + \int_{\partial\Omega} (g_{,\vartheta}\vartheta' + g_{,\vartheta_{,n}}\vartheta'_{,n} + (\nabla g\mathbf{n} + gH)\dot{D})\, \mathrm{d}(\partial\Omega) \\
& + (V_1 n_2 - V_2 n_1) g \,|_{\mathrm{B}}^{\mathrm{A}} \\
& - \int_\Omega (\nabla\vartheta'\nabla\eta + \nabla\vartheta\nabla\eta')\, \mathrm{d}\Omega - \int_{\partial\Omega} \nabla\vartheta\nabla\eta\dot{D} \mathrm{d}(\partial\Omega) \\
& - \int_{\partial\Omega} (q_{,\vartheta}\vartheta'\eta + q\eta' + (\nabla(q\eta)\mathbf{n} \\
& + q\eta H)\dot{D})\, \mathrm{d}(\partial\Omega) - (V_1 n_2 - V_2 n_1) q\eta \,|_{\mathrm{B}}^{\mathrm{A}} \\
& - \int_{\partial\Omega} (\vartheta'_{,n}\eta + \vartheta_{,n}\eta' + (\nabla(\vartheta_{,n}\eta)\mathbf{n} \\
& + \vartheta_{,n}\eta H)\dot{D})\, \mathrm{d}(\partial\Omega) - (V_1 n_2 - V_2 n_1)\vartheta_{,n}\eta \,|_{\mathrm{B}}^{\mathrm{A}}. \quad (21)
\end{aligned}$$

Using the relationship $\dot{z} = z' + \nabla z\mathbf{V}$ we get

$$\begin{aligned}
\delta\bar{J} = {} & \int_\Omega f_{,\vartheta}(\dot{\vartheta} - \nabla\vartheta\mathbf{V})\mathrm{d}\Omega + \int_{\partial\Omega} f\dot{D}\, \mathrm{d}(\partial\Omega) \\
& + \int_{\partial\Omega} (g_{,\vartheta}(\dot{\vartheta} - \nabla\vartheta\mathbf{V}) + g_{,\vartheta_{,n}}(\dot{\vartheta}_{,n} - \nabla\vartheta_{,n}\mathbf{V}) \\
& + (\nabla g\mathbf{n} + gH)\dot{D})\, \mathrm{d}(\partial\Omega) + (V_1 n_2 - V_2 n_1) g \,|_{\mathrm{B}}^{\mathrm{A}} \\
& + \int_\Omega (\nabla(\dot{\vartheta} - \nabla\vartheta\mathbf{V})\nabla\eta + \nabla\vartheta\nabla(\dot{\eta} - \nabla\eta\mathbf{v}))\, \mathrm{d}(\Omega) \\
& - \int_{\partial\Omega} \nabla\vartheta\nabla\eta\dot{D}\, \mathrm{d}(\partial\Omega) - \int_{\partial\Omega} (q_{,\vartheta}(\dot{\vartheta} - \nabla\vartheta\mathbf{V})\eta \\
& + q(\dot{\eta} - \nabla\eta\mathbf{V}) + (\nabla(q\eta)\mathbf{n} + q\eta H)\dot{D})\, \mathrm{d}(\partial\Omega) \\
& - (V_1 n_2 - V_2 n_1) q\eta \,|_{\mathrm{B}}^{\mathrm{A}} - \int_{\partial\Omega} ((\dot{\vartheta}_{,n} - \nabla\vartheta_{,n}\mathbf{V})\eta \\
& + \vartheta_{,n}(\dot{\eta} - \nabla\eta\mathbf{V}) + (\nabla(\vartheta_{,n}\eta)\mathbf{n} + \vartheta_{,n}\eta H)\dot{D})\, \mathrm{d}(\partial\Omega) \\
& - (V_1 n_2 - V_2 n_1)\vartheta_{,n}\eta \,|_{\mathrm{B}}^{\mathrm{A}}.
\end{aligned} \quad (22)$$

Several terms can be cancelled on account of the relation

$$\int_\Omega \nabla\vartheta\nabla\dot{\eta}\, \mathrm{d}\Omega + \int_{\partial\Omega} q\dot{\eta}\, \mathrm{d}(\partial\Omega) + \int_{\partial\Omega} \vartheta_{,n}\dot{\eta}\, \mathrm{d}(\partial\Omega) = 0. \quad (23)$$

If $\vartheta^* \in K$ is such that

$$\begin{aligned}
& \int_\Omega f_{,\vartheta}\dot{\vartheta}\, \mathrm{d}\Omega + \int_{\partial\Omega} g_{,\vartheta}\dot{\vartheta}\, \mathrm{d}(\partial\Omega) - \int_\Omega \nabla\dot{\vartheta}\nabla\vartheta^*\, \mathrm{d}(\Omega) \\
& - \int_{\partial\Omega} q_{,\vartheta}\dot{\vartheta}\vartheta^*\, \mathrm{d}(\partial\Omega) + \int_{\partial\Omega} g_{,\vartheta_{,n}}\dot{\vartheta}_n\, \mathrm{d}(\partial\Omega) \\
& - \int_{\partial\Omega} \dot{\vartheta}_{,n}\vartheta^*\, \mathrm{d}(\partial\Omega) = 0 \quad (24)
\end{aligned}$$

then equation (22) can be treated as a function of the domain variation $\mathbf{V}$ which in turn can be expressed as a function of the design variables.

If equation (24) holds for $\dot{\vartheta}$ then it hods for all $\xi \in K$. The adjoint equation can be presented as

$$\begin{aligned}
& \int_\Omega f_{,\vartheta}\xi\, \mathrm{d}\Omega + \int_{\partial\Omega} g_{,\vartheta}\xi\, \mathrm{d}(\partial\Omega) - \int_\Omega \nabla\xi\nabla\vartheta^*\, \mathrm{d}(\Omega) \\
& - \int_{\partial\Omega} q_{,\vartheta}\xi\vartheta^*\, \mathrm{d}(\partial\Omega) + \int_{\partial\Omega} g_{,\vartheta_{,n}}\xi_{,n}\, \mathrm{d}(\partial\Omega) \\
& - \int_{\partial\Omega} \xi_{,n}\vartheta^*\, \mathrm{d}(\partial\Omega) = 0. \quad (25)
\end{aligned}$$

The sensitivity equation takes the form

$$\delta\bar{J} = -\int_\Omega f_{,\vartheta}\nabla\vartheta\mathbf{V}\, \mathrm{d}\Omega + \int_{\partial\Omega} f\dot{D}\, \mathrm{d}(\partial\Omega)$$

$$-\int_{\partial\Omega}(g_{,\vartheta}\nabla\mathbf{V}+g_{,\vartheta_{,n}}\nabla\vartheta_{,n}\mathbf{V}$$
$$-(\nabla g\mathbf{n}+gH)\dot{D})\,\mathrm{d}(\partial\Omega)+(V_1n_2-V_2n_1)g\,|_{\mathrm{B}}^{\mathrm{A}}$$
$$+\int_{\Omega}(\nabla(\nabla\vartheta\mathbf{V})\nabla\eta+\nabla\vartheta\nabla(\nabla\eta\mathbf{V}))\,\mathrm{d}\Omega$$
$$-\int_{\partial\Omega}\nabla\vartheta\nabla\eta\dot{D}\,\mathrm{d}(\partial\Omega)$$
$$+\int_{\partial\Omega}(q_{,\vartheta}\nabla\vartheta\mathbf{V}\eta+q\nabla\eta\mathbf{V}-(\nabla(q\eta)\mathbf{n}$$
$$+q\eta H)\dot{D})\,\mathrm{d}(\partial\Omega)-(V_1n_2-V_2n_1)q\eta\,|_{\mathrm{B}}^{\mathrm{A}}$$
$$+\int_{\partial\Omega}(\nabla\vartheta_{,n}\eta\mathbf{V}+\vartheta_{,n}\nabla\vartheta\mathbf{V}-(\nabla(\vartheta_{,n}\eta)\mathbf{n}$$
$$+\vartheta_{,n}\eta H)\dot{D})\,\mathrm{d}(\partial\Omega)-(V_1n_2-V_2n_1)\vartheta_{,n}\eta\,|_{\mathrm{B}}^{\mathrm{A}}. \tag{26}$$

Variational forms of equations (19) and (25) can be solved numerically by FEM for ϑ and ϑ^* and the solutions used to calculate the sensitivity in equation (26). Applying the Green's formula inversely to the third term of equation (25) we get

$$\int_{\Omega}f_{,\vartheta}\xi\,\mathrm{d}\Omega+\int_{\partial\Omega}g_{,\vartheta}\xi\,\mathrm{d}(\partial\Omega)+\int_{\Omega}\nabla^2\vartheta^*\xi\,\mathrm{d}\Omega$$
$$-\int_{\partial\Omega}\frac{\partial\vartheta^*}{\partial n}\xi\,\mathrm{d}(\partial\Omega)-\int_{\partial\Omega}q_{,\vartheta}\vartheta^*\xi\,\mathrm{d}(\partial\Omega)$$
$$+\int_{\partial\Omega}g_{,\vartheta_{,n}}\xi_{,n}\,\mathrm{d}(\partial\Omega)-\int_{\partial\Omega}\vartheta^*\xi_{,n}\,\mathrm{d}(\partial\Omega)=0. \tag{27}$$

Rearranging the terms we obtain

$$\int_{\Omega}(\nabla^2\vartheta^*+f_{,\vartheta})\xi\,\mathrm{d}\Omega+\int_{\partial\Omega}\left(g_{,\vartheta}-q_{,\vartheta}\vartheta^*-\frac{\partial\vartheta^*}{\partial n}\right)\xi\,\mathrm{d}(\partial\Omega)$$
$$+\int_{\partial\Omega}(g_{,\vartheta_{,n}}-\vartheta^*)\xi_{,n}\,\mathrm{d}(\partial\Omega)=0. \tag{28}$$

Therefore the adjoint system reduces to

$$\nabla^2\vartheta^*+f_{,\vartheta}=0 \quad \text{in } \Omega \tag{29}$$
$$\frac{\partial\vartheta^*}{\partial n}=-q_{,\vartheta}\vartheta^*+g_{,\vartheta} \quad \text{in } \Omega q \tag{30}$$
$$\vartheta^*=g_{,\vartheta_{,n}} \quad \text{in } \Omega_{\mathrm{T}}. \tag{31}$$

EXAMPLE PROBLEM

As an example we take the thermal diffuser shown schematically in Fig. 1. The problem is treated as axisymmetric with the x-axis taken as the axis of symmetry.

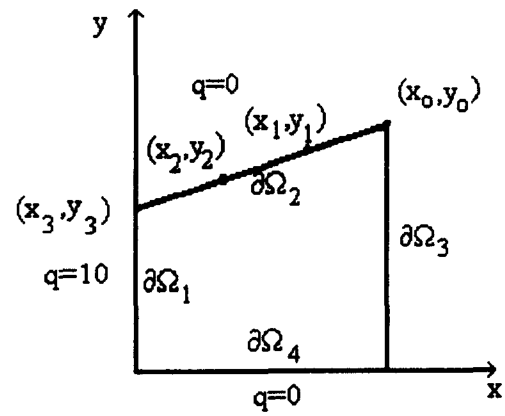

Fig. 1. Diffuser under consideration.

The system is described by the following boundary value problem:

$$\nabla(k\nabla T)=0 \quad \text{in } \Omega$$
$$q=0 \quad \text{on } \partial\Omega_2\cup\partial\Omega_4$$
$$q=10\,\mathrm{W\,m^{-2}} \quad \text{on } \partial\Omega_1$$
$$k=200(1+0.0T)\,\mathrm{W\,m^{-2}\,{}^\circ C}$$
$$(x_0,y_0)=(0.1\,\mathrm{m},0.1\,\mathrm{m})$$
$$(x_3,y_3)=(0\,\mathrm{m},0.05\,\mathrm{m}).$$

The cost functional is assumed as

$$J=\int_{\partial\Omega_3}(T-T_{\mathrm{R}})^2\,\mathrm{d}(\partial\Omega),$$

where T_{R} is the given temperature of 50°C.

In practice the thermal diffuser problem leads to optimization of the shape of the domain Ω, which gives a uniform temperature distribution on $\partial\Omega_3$ under the maximum area constraint. Thus, the sensitivity analysis of the functional J has here a practical interpretation. For the shape sensitivity analysis $\partial\Omega_1$ and $\partial\Omega_4$ are assumed to be fixed, $\partial\Omega_3$ is allowed to move horizontally and $\partial\Omega_2$ is allowed to change shape. Let $\partial\Omega_2$ be approximated by the cubic function

$$y=\sum_{n=0}^{3}a_nx^n$$

which has to be satisfied by the coordinates of the four modal points (x_i,y_i), $i=0,1,2,3$; such that $x_1=2/3x_0$, $x_2=1/3x_0$, $x_3=0$.

The design variable vector for the problem can be defined as $\mathbf{d}=[d_0,d_1,d_2]=[y_0,y_1,y_2]$. We have

$$y=\sum_{m=0}^{2}\sum_{n=0}^{3}c_{nm}x^nd_m+\sum_{n=0}^{3}c_{n3}x^ny_3$$

and the velocity vector field on $\partial\Omega_2$ is expressed as

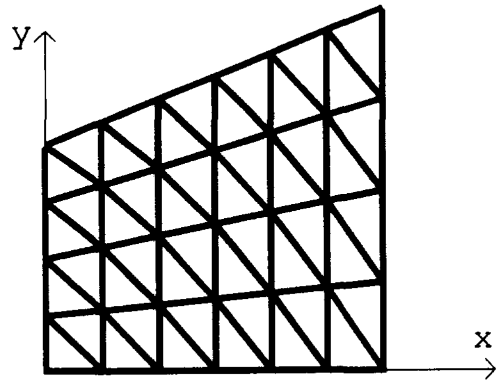

Fig. 2. Finite element mesh.

$$V = \sum_{m=0}^{2} \sum_{n=0}^{3} c_{nm} x^n \delta d_m,$$

where

$$[c_{nm}] = \begin{bmatrix} 1 & x_0 & x_0^2 & x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \end{bmatrix}$$

The variables in the sensitivity equation are given as

$$n_1 = \frac{y'(x)}{\sqrt{1+(y'(x))^2}}$$

$$n_2 = \frac{-1}{\sqrt{1+(y'(x))^2}}$$

$$H = \frac{|y''(x)|}{(1+(y'(x))^2)^{3/2}}.$$

The finite element model of the diffuser is shown in Fig. 2.

The results of the sensitivity analysis are given in Table 1. Two separate problems are considered: the nominal design is given by $\mathbf{d} = [0.1, 0.0833, 0.0666]$ in the problem A and by $\mathbf{d} = [0.09, 0.0823, 0.0543]$ in the problem B. The values J_1 of the cost functional in Table 1 correspond to the nominal designs $\mathbf{d}^0$, while the values J_2 correspond to successive values of the cost functional for the design vector modified by increasing each of its components in turn by 1% (i.e. $\mathbf{d}^1$, $\mathbf{d}^2$, $\mathbf{d}^3$ have been obtained by increasing the first, second and third component in $\mathbf{d}^0$ by 1%, respectively), ΔJ stands for the difference between J_2 and J_1, while J' is the sensitivity gradient obtained by the technique proposed in this paper. On the whole the results by both of the methods are close to each other. Existing differences between ΔJ and J' can be attributed to errors inherent in the finite difference evaluation of sensitivity.

Table 1. Sensitivity analysis for the thermal diffusion problem

Problem	Design	J_1	J_2	$\Delta J = J_2 - J_1$	J'
	$\mathbf{d}^0$	33.256			
A	$\mathbf{d}^1$		32.799	−0.457	−0.511
	$\mathbf{d}^2$		30.828	−2.428	−2.540
	$\mathbf{d}^3$		31.717	−1.538	−1.559
	$\mathbf{d}^0$	108.550			
B	$\mathbf{d}^1$		109.094	0.545	0.448
	$\mathbf{d}^2$		101.081	−7.469	−7.763
	$\mathbf{d}^3$		105.141	−3.409	−3.492

CONCLUSIONS

(1) The Kirchhoff transformation has been proved to be a useful tool in developing an effective technique for shape design sensitivity assessment in nonlinear steady-state heat conduction problems.

(2) Implementation of the method in the standard finite element program is straightforward; the method can thus be used for optimization of problems of industrial significance.

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